What Survives

A Validated Mathematical Reconstruction of the S-I-C-T Framework

Conjunctive feasibility, scale-invariant bottleneck margins, conditional dynamical stability — and an open call for independent testing

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Abstract. The Structure–Information–Cohesion–Transformation (S-I-C-T) framework proposes four coordinates for diagnosing complex adaptive systems. An earlier working draft replaced its additive viability condition $V = S + C - I - T \geq 0$ with a conjunctive, weakest-link margin. This paper subjects that reconstruction to a line-by-line validation: every theorem, counterexample and numerical value is re-derived by hand and checked by a 41-test replication suite (Appendix A). No stated theorem is false. Four statements needed repair or sharpening: a quartic characteristic polynomial valid only for 2×2 blocks, an unstated constraint qualification in the invariance theorem, a domain restriction on the critical-slowing-down bridge, and a Gershgorin stability bound that the paper's own Lyapunov bound strictly dominates. We add four results. First, the normalized margin W_N is an exact monotone transform of the classical bottleneck load ratio $\rho^* = \max(I/S,\ T/C)$, so its ranking performance equals that of bottleneck utilization by construction. Second, an observable error bound from raw measurement error. Third, a scale-free bound under relative error. Fourth, in the linear coupled model a positive weighted *additive* margin is necessary and a coupling-adjusted *weakest-link* margin is sufficient for stability. The framework survives as a conditional, non-compensatory capacity–load diagnostic, not a universal stability law; any novelty lies in cross-domain operationalization, not in new mathematics. Because that claim is empirical, we close with a preregistration-ready protocol and an open call for independent proof review, replication, counterexamples and data partnerships.

Keywords: complex adaptive systems; S-I-C-T framework; weakest-link aggregation; bottleneck utilization; viability theory; positive invariance; Lyapunov stability; early-warning signals; measurement theory; falsification.

Open call — help us test, and try to break, S-I-C-T

We are looking for **proof reviewers**, **replicators**, **counterexample hunters**, **data partners** with paired capacity–load time series, **testbed operators** (agentic AI, SRE, service operations) and **statisticians** for preregistration. Null and negative results will be reported with credit. Details, deliverables and recognition: Section 17. Contact: miklosroth@rothcomplexity.org, subject line "S-I-C-T TEST — [your role]".

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1. Introduction and validation summary

Complex adaptive systems combine structural constraints, information-processing demands, coordination mechanisms and transformational pressures. Similar vocabulary appears across biology, engineering, economics and organizations, but shared vocabulary does not imply shared mathematics. The S-I-C-T framework proposes four observed coordinates

$$\mathbf{x} = (S, I, C, T)^\top, \tag{1}$$

where S is structural (processing) capacity, I is information-processing load, C is coordination capacity and T is transformational load. Its original diagnostic was additive,

$$V = S + C - I - T, \tag{2}$$

which lets a surplus in one channel hide a deficit in another. The working draft therefore proposed the conjunctive margin

$$W = \min(S - I,\ C - T). \tag{3}$$

What this version does. Before asking anyone to spend effort on empirical tests, we audited the draft as a referee would. Each claim was re-derived and then attacked numerically: randomized falsification over 2×10^5 draws for algebraic identities, eigenvalue checks for stability statements, stochastic simulation for the Ornstein–Uhlenbeck and queueing claims, and exact recomputation of every number printed in a figure. The suite and its output are reproduced in Appendices A and B. Table 1 records the verdict.

The four epistemic badges follow the Roth Complexity Lab convention: ESTABLISHED (proved, conditional only on stated hypotheses), GROUNDED (follows from established theory once a stated modeling assumption is accepted), HEURISTIC (a plausible bridge or analogy, not derived), SPECULATIVE (an open empirical claim). NOT SUPPORTED marks claims that fail.

Table 1. Validation verdict for every claim in the reconstruction.

Claim	Verdict after validation	Status
Algebra and geometry of the margin		
Thm 1 — conjunctive equivalence	Holds exactly.	ESTABLISHED
Cor 1 — additive relaxation, masking	Holds. For i.i.d. standard normal margins, masking occurs with probability exactly $1/4$.	ESTABLISHED
Thm 2 — bottleneck decomposition	Holds exactly.	ESTABLISHED
Thm 3 — boundedness, sign	Holds for nonnegative coordinates and positive denominators.	ESTABLISHED
Thm 4 — pairwise scale invariance	Holds. Fails under independent per-coordinate rescaling (sign flips in 19.7% of test draws).	ESTABLISHED
Log form (8) — equal AUROC	Holds.	ESTABLISHED
Prop 1 — $W_N = (1-\rho^*)/(1+\rho^*)$	New: rank-equivalent to the bottleneck load ratio.	NEW ESTABLISHED
Thm 5 — monotonicity	Holds.	ESTABLISHED
Thm 6 — robustness (margin errors)	Holds (min is 1-Lipschitz in ℓ_∞).	ESTABLISHED
Props 2–3 — raw-error bounds	New: observable absolute bound; scale-free relative bound.	NEW ESTABLISHED

Claim	Verdict after validation	Status
Thm 7 — signed ℓ_∞ distance	Holds; confirmed by brute-force projection.	ESTABLISHED
Thm 8 — soft-min bounds	Holds; the upper inequality is strict.	ESTABLISHED
Dynamics		
Thm 9 — positive invariance	Holds once closedness, uniqueness and a constraint qualification are explicit; it is an application of Nagumo's theorem.	CORRECTED GROUNDED
Thm 10 — Routh–Hurwitz	Holds exactly.	ESTABLISHED
Cor 2 — Gershgorin threshold	True but strictly dominated by the Lyapunov bound (Prop 4).	ESTABLISHED
Prop 4 — Lyapunov dominates	New: $\lvert u+v\rvert/2 \le \max(\lvert u\rvert,\lvert v\rvert)$.	NEW ESTABLISHED
Prop 5 — additive necessary, conjunctive sufficient	New: clarifies the roles of V and W inside the model.	NEW ESTABLISHED
Thm 11 — exponential stability	Holds; confirmed on a simulated time-varying system.	ESTABLISHED
§11.1–11.2 counterexamples	Verified: spectra $\{1,-3\}$ and $-0.4\pm0.8i$.	ESTABLISHED
§11.3 basin blindness	Was schematic; now constructive (Prop 6).	CORRECTED ESTABLISHED
§11.4 non-identifiability	True but elementary; restated as Prop 7.	ESTABLISHED
§12 undamped bipartite system	The quartic holds only for 2×2 blocks; general form $\det(q^2I-BC)$.	CORRECTED ESTABLISHED
§13 OU formulas	Exact; Monte Carlo agrees.	ESTABLISHED
§13 bridge $\kappa = kW$	An extra assumption, defined only for $W>0$.	CORRECTED HEURISTIC
§14 queueing interpretation	Exact for M/M/1; extends to open Jackson networks (Prop 8).	ESTABLISHED
§15.5 data-processing limit	Holds.	ESTABLISHED
Scientific claims		
S-I-C-T as a universal stability law	Rejected by the counterexamples.	NOT SUPPORTED
Raw minimum is unit-independent	False.	NOT SUPPORTED
Normalization makes heterogeneous measures commensurable	False; commensurability is a prior measurement requirement.	NOT SUPPORTED
Link to Ashby's law of requisite variety	Structural analogy, not derivation.	HEURISTIC
Predicts critical transitions; beats classical indicators	Untested. Against bottleneck utilization, ranking gain is zero by Prop 1.	SPECULATIVE
Cross-domain transfer of a fixed operationalization	Untested.	SPECULATIVE
Mathematical novelty	Not established; the components are standard.	NOT SUPPORTED

Bottom line. The mathematics is sound but mostly standard. The strongest defensible object is a dimensionless, non-compensatory capacity–load diagnostic that gains dynamical meaning only under explicit structural assumptions. Proposition 1 sharpens the empirical question: *the formula is classical; the open question is whether S-I-C-T picks the right capacity–load pairs in domains where nobody measures them yet.*

2. Epistemic and Mathematical Foundations

2.1 Three logically independent levels

Level	Mathematical question	Typical instrument
I. Diagnostic feasibility	Are specified capacity constraints satisfied?	$W_N \geq 0$
II. Dynamical stability	Do perturbations decay under a specified evolution law?	Jacobian, Lyapunov function
III. Predictive validity	Does the indicator improve out-of-sample forecasts?	Preregistered Δ AUROC

A result at one level does not establish a result at another. In particular, a positive diagnostic margin cannot establish asymptotic stability without a dynamical model (Section 11).

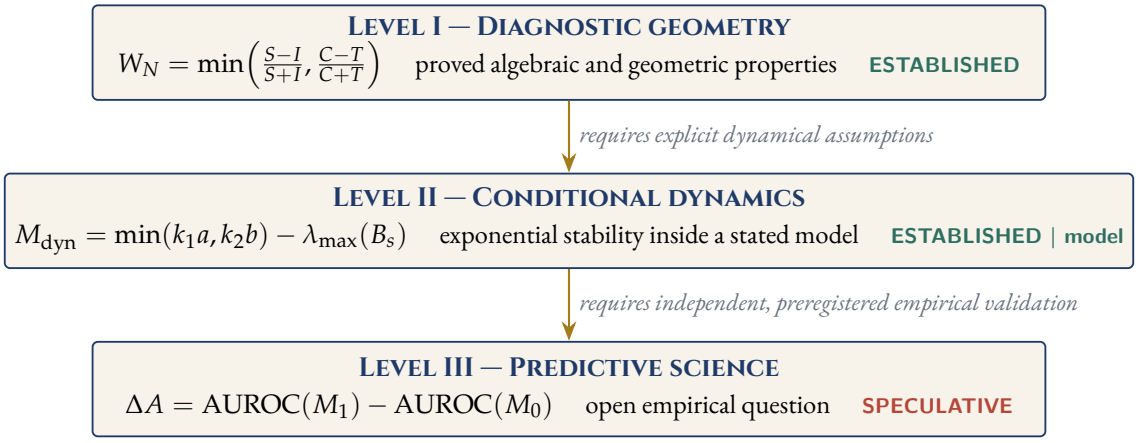

Figure 1. The three-level architecture. Arrows are not implications: each step requires new assumptions or new evidence.

2.2 Operational assumptions

Let X be the underlying state space with observation maps $S, I, C, T : X \to \mathbb{R}_{\geq 0}$. The following are modeling assumptions, not established properties of complex systems.

Axiom A1 (Pairwise commensurability). S and I are measured on a common ratio scale with a common unit; likewise C and T.

Axiom A2 (Directional monotonicity). Raising a capacity, all else fixed, cannot lower feasibility; raising a load cannot raise it.

Axiom A3 (Conjunctive necessity). Both capacity constraints are independently required for the admissibility notion being modeled.

Axiom A4 (A priori specification). Measurement maps, thresholds and horizons are fixed before access to held-out outcomes.

Axiom A5 (Conditional dynamics). No stability interpretation is used unless an explicit evolution law links the margins to perturbation dynamics.

3. Conjunctive Feasibility

Define the capacity margins and the admissible set

$$a = S - I, \qquad b = C - T, \qquad \mathcal{K} = \{x \in X : a(x) \geq 0,\, b(x) \geq 0\}. \tag{4}$$

Theorem 1 (Conjunctive equivalence). **ESTABLISHED** *For every* $x \in X$, $x \in \mathcal{K} \iff W(x) \geq 0$.

Proof. $W = \min(a, b)$, and the minimum of two reals is nonnegative if and only if both are. ■

Corollary 1 (Additive relaxation). **ESTABLISHED** $W \geq 0 \Rightarrow V \geq 0$, *since* $a + b \geq 2\min(a, b)$. *The converse fails:* $(S, I, C, T) = (0.9, 0.2, 0.3, 0.8)$ *gives* $V = 0.2$ *but* $W = -0.5$.

The set where additive aggregation hides a deficit is

$$\mathcal{M} = \{(a, b) : a + b \geq 0,\ \min(a, b) < 0\}. \tag{5}$$

Remark 1 (How large is $\mathcal{M}$?). If a, b are i.i.d. symmetric continuous, $\Pr[\mathcal{M}] = \Pr[a + b \geq 0] - \Pr[a \geq 0, b \geq 0] = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$. In this neutral reference case, one quarter of all states that pass the additive test fail the conjunctive one (test 3 in Appendix A). This is an illustration, not an empirical base rate.

Theorem 2 (Bottleneck decomposition). **ESTABLISHED** *For all real* a, b,

$$W = \frac{V - |a - b|}{2}, \qquad \textit{hence} \qquad W \geq 0 \iff V \geq |a - b|. \tag{6}$$

Proof. Use $\min(a, b) = \frac{1}{2}\big(a + b - |a - b|\big)$ with $a + b = V$. ■

The additive surplus must cover the imbalance between the two margins. This is an exact algebraic form of non-compensatory feasibility.

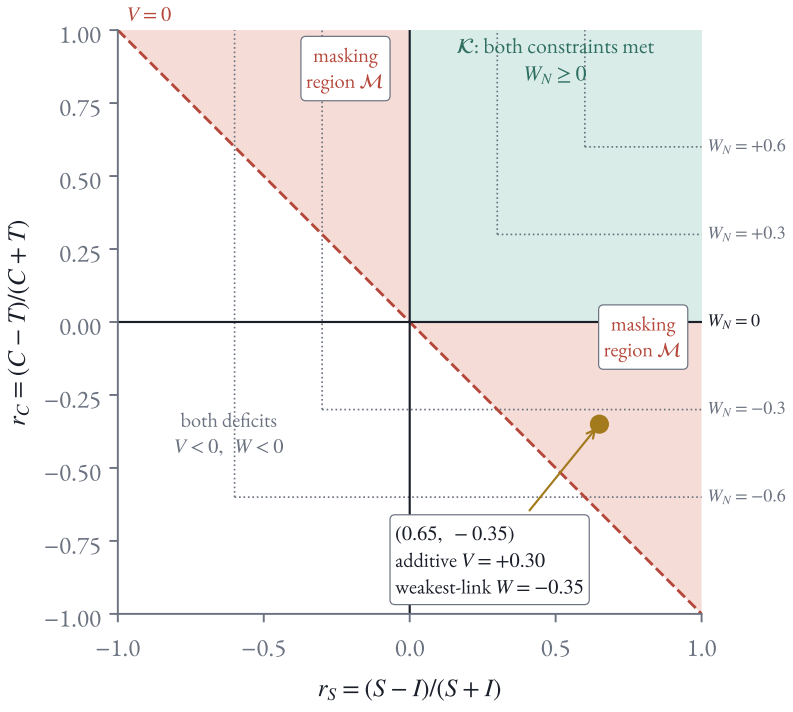

Figure 2. Normalized viability space. Shaded: the admissible region $\mathcal{K}$. Hatched: the masking region $\mathcal{M}$, where the additive indicator is nonnegative but a constraint is violated. Dotted L-shapes: level sets of W_N. The marked state $(0.65, -0.35)$ has additive margin $+0.30$ and weakest-link margin -0.35.

4. Dimensionless normalization and its classical identity

Raw differences depend on units. For $S + I > 0$ and $C + T > 0$ define

$$r_S = \frac{S - I}{S + I}, \qquad r_C = \frac{C - T}{C + T}, \qquad W_N = \min(r_S, r_C). \tag{7}$$

Theorem 3 (Boundedness and sign preservation). ESTABLISHED $W_N \in [-1, 1]$, *and* $W_N \geq 0 \iff (S \geq I) \wedge (C \geq T)$.

Proof. For $p, q \geq 0$ with $p + q > 0$, $|p - q| \leq p + q$, so $(p - q)/(p + q) \in [-1, 1]$ and has the sign of $p - q$. Apply this to both pairs and take the minimum. ■

Theorem 4 (Pairwise scale invariance). ESTABLISHED *For all* $\alpha, \beta > 0$,

$$W_N(\alpha S, \alpha I, \beta C, \beta T) = W_N(S, I, C, T).$$

Proof. $(\alpha S - \alpha I)/(\alpha S + \alpha I) = (S - I)/(S + I)$, and likewise for the second pair. ■

This invariance does *not* extend to independent rescaling of the four coordinates. In the test suite, independent log-normal rescaling flipped the sign of W_N in 19.7% of draws. Normalization also does not make unrelated measurements commensurable: Axiom A1 must hold *before* normalization (cf. the meaningfulness literature on aggregation over independent scales [14, 15]).

4.1 Logarithmic form

For strictly positive coordinates let $\ell_S = \log(S/I)$, $\ell_C = \log(C/T)$. Then $r_S = \tanh(\ell_S/2)$, and since $\tanh$ is strictly increasing,

$$W_N = \tanh\left(\tfrac{1}{2}L\right), \qquad L = \min\left(\log \tfrac{S}{I}, \log \tfrac{C}{T}\right). \tag{8}$$

So W_N and L induce identical rankings, and their AUROC values coincide on the same observations.

4.2 The classical identity

Proposition 1 (Rank equivalence with the bottleneck load ratio). NEW ESTABLISHED *Let* $S, C > 0$ *and* $I, T \geq 0$, *and define the load ratios* $\rho_S = I/S$, $\rho_C = T/C$ *and* $\rho^* = \max(\rho_S, \rho_C)$. *Then*

$$\boxed{W_N = \varphi(\rho^*) = \frac{1 - \rho^*}{1 + \rho^*}}, \qquad \varphi(x) = \frac{1 - x}{1 + x} \text{ strictly decreasing on } [0, \infty). \tag{9}$$

Proof. $r_S = (S - I)/(S + I) = (1 - \rho_S)/(1 + \rho_S) = \varphi(\rho_S)$, and likewise $r_C = \varphi(\rho_C)$. For a decreasing φ, $\min\{\varphi(\rho_S), \varphi(\rho_C)\} = \varphi(\max\{\rho_S, \rho_C\})$. ■

Consequences. (i) W_N, $L = -\log \rho^*$ and ρ^* order observations identically, so every rank-based metric is identical: AUROC, Spearman correlation, top-k precision. (ii) $W_N \geq 0 \iff \rho^* \leq 1$. (iii) In service systems ρ^* is the *bottleneck utilization*, one of the oldest operational indicators. As a single score, the normalized S-I-C-T margin is therefore a re-expression of bottleneck utilization. (iv) Comparing W_N against ρ^* as single scores must give $\Delta\,\text{AUROC} = 0$ exactly. A nonzero difference signals a pipeline error. We use this as a built-in canary in Section 15.

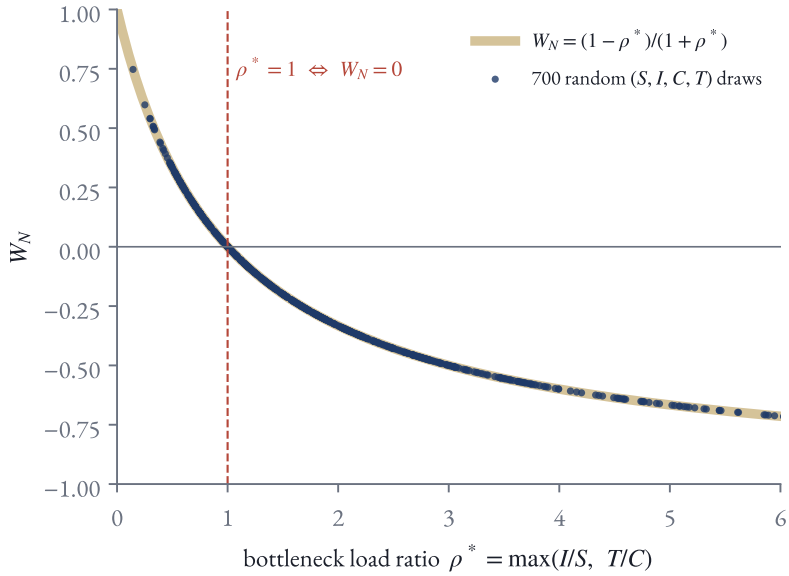

Figure 3. Seven hundred random (S, I, C, T) draws collapse exactly onto $W_N = (1-\rho^*)/(1+\rho^*)$. The crossing $W_N = 0$ is the bottleneck reaching full load, $\rho^* = 1$.

5. Monotonicity and Measurement Robustness

Theorem 5 (Capacity–load monotonicity). **ESTABLISHED** *For* $r(p,q) = (p-q)/(p+q)$,

$$\frac{\partial r}{\partial p} = \frac{2q}{(p+q)^2} \geq 0, \qquad \frac{\partial r}{\partial q} = -\frac{2p}{(p+q)^2} \leq 0. \tag{10}$$

Since the minimum is coordinatewise nondecreasing, W_N *is nondecreasing in* S, C *and nonincreasing in* I, T *(Axiom A2).*

Theorem 6 (Robustness to margin errors). **ESTABLISHED** *If* $|\hat{r}_S - r_S| \leq \epsilon_S$ *and* $|\hat{r}_C - r_C| \leq \epsilon_C$, *then*

$$|\widehat{W}_N - W_N| \leq \max(\epsilon_S, \epsilon_C). \tag{11}$$

Proof. The minimum is 1-Lipschitz in the maximum norm: $|\min(x_1, x_2) - \min(y_1, y_2)| \leq \max_i |x_i - y_i|$. ∎

Errors usually live in the raw coordinates, not in the normalized margins. The next two results close that gap.

Proposition 2 (Observable bound under absolute raw error). **NEW** **ESTABLISHED** *Let true values* $p, q \geq 0$ *with* $p + q > 0$ *be observed as* $\hat{p} = p + \delta_p$, $\hat{q} = q + \delta_q$ *with* $|\delta_p|, |\delta_q| \leq \eta$ *and* $\hat{p} + \hat{q} > 0$. *Then*

$$|r(\hat{p}, \hat{q}) - r(p, q)| \leq \frac{2\eta}{\hat{p} + \hat{q}}. \tag{12}$$

Proof. A direct computation gives $r(\hat{p}, \hat{q}) - r(p, q) = \dfrac{2(q\delta_p - p\delta_q)}{(p+q)(\hat{p}+\hat{q})}$. Since $p, q \geq 0$, $|q\delta_p - p\delta_q| \leq \eta(p+q)$, which yields (12). ∎

The bound depends only on observed quantities, so it can be computed per observation. With (11), $|\widehat{W}_N - W_N| \leq \max\{2\eta_S/(\hat{S}+\hat{I}),\ 2\eta_C/(\hat{C}+\hat{T})\}$. It also quantifies the warning in the draft: the bound blows up as the observed denominators approach zero.

Proposition 3 (Scale-free bound under relative error). NEW ESTABLISHED *Let* $p, q > 0$ *be observed as* $\hat{p} = p(1+e_p)$, $\hat{q} = q(1+e_q)$ *with* $|e_p|, |e_q| \le e < 1$. *Then*

$$|r(\hat{p},\hat{q}) - r(p,q)| \le -\log(1-e). \tag{13}$$

Proof. $r = \tanh(\ell/2)$ with $\ell = \log p - \log q$, and $\tanh$ is 1-Lipschitz, so $|\Delta r| \le \frac{1}{2}|\Delta \ell| \le \frac{1}{2}\big(|\log(1+e_p)| + |\log(1+e_q)|\big) \le -\log(1-e)$, because $|\log(1+t)| \le -\log(1-e)$ for $|t| \le e$. ∎

For 10% relative error the margin error is at most 0.1054, whatever the units or magnitudes. With bound $\epsilon = \max(\epsilon_S, \epsilon_C)$ from (12) or (13):

Observed condition	Diagnostic interpretation
$\widehat{W}_N > \epsilon$	Both true margins positive: robust admissibility
$\widehat{W}_N < -\epsilon$	At least one true margin negative: robust deficit
$\lvert\widehat{W}_N\rvert \le \epsilon$	Sign not identifiable at this error level

These statements are only as good as the error bounds behind them.

6. Geometric interpretation

In the normalized plane $z = (r_S, r_C) \in [-1,1]^2$ let $K = [0,\infty)^2$ and define the signed ℓ_∞ distance

$$d_K^{\pm}(z) = \begin{cases} d_\infty(z, K^c), & z \in K, \\ -d_\infty(z, K), & z \notin K. \end{cases} \tag{14}$$

Theorem 7 (Signed-distance representation). ESTABLISHED $W_N(z) = d_K^{\pm}(z)$.

Proof. For $z \in K$, the cheapest way to leave K in the max norm is to push the smaller coordinate below zero, at cost $\min(r_S, r_C)$. For $z \notin K$, the ℓ_∞ projection raises each negative coordinate to zero, so $d_\infty(z,K) = \max(0, -r_S, -r_C) = -\min(r_S, r_C)$ because at least one coordinate is negative. ∎

W_N is therefore a signed feasibility margin in observation space. It is not, in general, a distance to a dynamical basin boundary in state space (Proposition 6).

7. Smooth approximation

The minimum is not differentiable where $r_S = r_C$. For gradient-based work use

$$W_\tau = -\tau \log\big(e^{-r_S/\tau} + e^{-r_C/\tau}\big), \qquad \tau > 0. \tag{15}$$

Theorem 8 (Uniform approximation). ESTABLISHED $W_N - \tau\log 2 \le W_\tau < W_N$, *hence* $W_\tau \to W_N$ *as* $\tau \to 0^+$.

Proof. With $m = \min$ and $M = \max$ of the two margins, $W_\tau = m - \tau\log\big(1 + e^{-(M-m)/\tau}\big)$, and $0 < \log(1+e^{-s}) \le \log 2$ for $s \ge 0$. ∎

The approximation is conservative: $W_\tau \ge 0$ implies $W_N > 0$. The converse can fail near the boundary. At $\tau = 0.15$ the maximum error is $0.15\log 2 = 0.1040$.

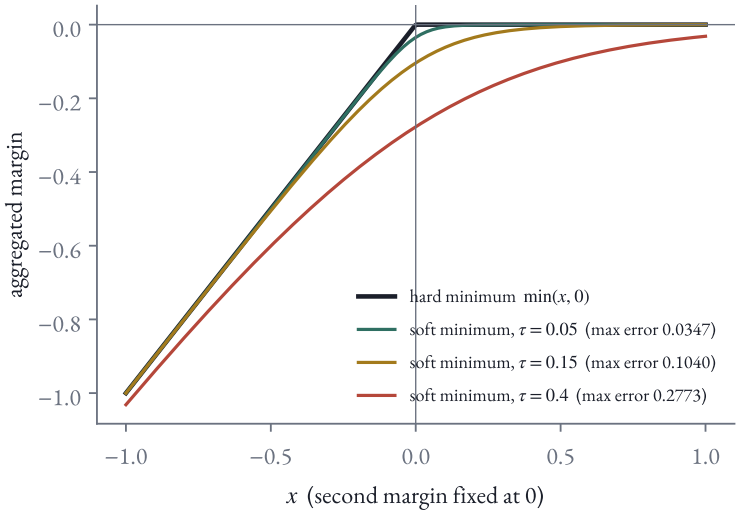

Figure 4. Hard and soft minimum with the second margin fixed at zero. The gap never exceeds $\tau \log 2$.

8. From feasibility to positive invariance

Let $\dot{x} = f(x)$ and $h_1 = S - I, h_2 = C - T$, so $\mathcal{K} = \{x : h_1(x) \geq 0,\ h_2(x) \geq 0\}$.

Theorem 9 (Conditional positive invariance). CORRECTED GROUNDED *Assume (i) $h_1, h_2 \in C^1$, so $\mathcal{K}$ is closed; (ii) f is locally Lipschitz, so forward solutions are unique; (iii) the Mangasarian–Fromovitz constraint qualification holds on $\partial\mathcal{K}$: at each boundary point some w satisfies $\nabla h_i(x)^\top w > 0$ for every active i. If*

$$\nabla h_i(x)^\top f(x) \geq 0 \qquad \text{for every active constraint } i \text{ at every } x \in \partial\mathcal{K}, \tag{16}$$

then $\mathcal{K}$ is positively invariant: solutions starting in $\mathcal{K}$ stay in $\mathcal{K}$ on their forward interval of existence.

Proof. Under (iii) the Bouligand tangent cone of $\mathcal{K}$ equals the linearized cone $\{w : \nabla h_i^\top w \geq 0,\ i \text{ active}\}$, so the hypothesis places $f(x)$ in the tangent cone at every boundary point. Nagumo's theorem [16, 4], with uniqueness from (ii), gives positive invariance. ■

Correction. The draft required only "an appropriate regularity condition". Without a constraint qualification the linearized cone can be strictly larger than the tangent cone at corners, and the conclusion can fail. The result is a standard application of viability theory [3], not a new law. Three notions remain distinct: $W_N \geq 0$ is instantaneous feasibility; positive invariance is persistence in $\mathcal{K}$; asymptotic stability is convergence to an attractor.

9. Conditional local stability

Consider a local perturbation $\dot{y} = Jy$ with

$$J = \begin{pmatrix} -k_1 a & u \\ v & -k_2 b \end{pmatrix}, \qquad k_1, k_2 > 0, \tag{17}$$

where u, v are cross-channel couplings.

Theorem 10 (Exact stability conditions). ESTABLISHED *J is Hurwitz if and only if $k_1 a + k_2 b > 0$ and $k_1 k_2 ab - uv > 0$.*

Proof. $p(\lambda) = \lambda^2 + (k_1 a + k_2 b)\lambda + (k_1 k_2 ab - uv)$. A real monic quadratic has both roots in the open left half-plane exactly when both coefficients are positive. ■

Corollary 2 (Gershgorin threshold). **ESTABLISHED** *With* $\rho = \max(|u|, |v|)$ *and* $k_{\min} = \min(k_1, k_2)$, *if* $W > \rho / k_{\min}$ *then* J *is Hurwitz.*

Proof. $k_1 a \geq k_{\min} W > \rho \geq |u|$ and $k_2 b > |v|$: both Gershgorin discs lie in the open left half-plane. ■

Proposition 4 (The Lyapunov bound dominates). **NEW** **ESTABLISHED** *For constant coupling* $B = \left(\begin{smallmatrix} 0 & u \\ v & 0 \end{smallmatrix}\right)$, $\lambda_{\max}(B_s) = |u + v|/2 \leq \max(|u|, |v|)$. *Hence the condition of Cor. 2 implies the condition of Theorem 11,* $\min(k_1 a, k_2 b) > |u + v|/2$, *which also certifies monotone Euclidean contraction. The converse fails: for* $a = b = k_1 = k_2 = 1$ *and skew coupling* $u = -v = 5$, *Theorem 11 certifies stability (eigenvalues* $-1 \pm 5i$*) while Gershgorin requires* $W > 5$.

In the test suite, 3,171 random systems were certified by the Lyapunov bound but not by Gershgorin, and none the other way round (test 30).

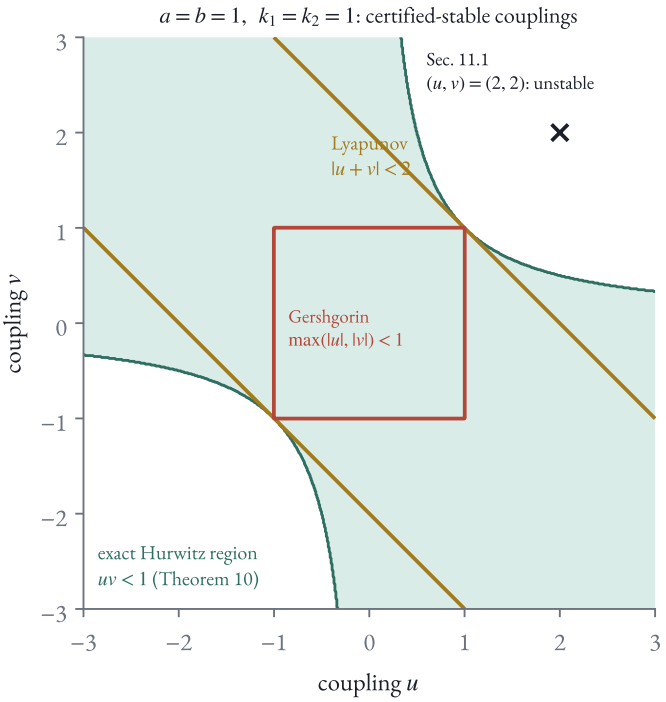

Figure 5. For $a = b = 1, k_1 = k_2 = 1$: the exact stability region $uv < 1$ (shaded) contains the Lyapunov certificate $|u + v| < 2$, which contains the Gershgorin square. The draft's counterexample $(u, v) = (2, 2)$ lies outside all three.

Proposition 5 (Additive necessity, conjunctive sufficiency). **NEW** **ESTABLISHED** *In model* (17)*: (a) the weighted* additive *margin* $V_k = k_1 a + k_2 b > 0$ *is* necessary *for stability; (b) it is not sufficient (§11.1:* $V_k = 2$, *unstable); (c)* $W \geq 0$ *is not necessary (§11.2:* $W = -0.2$, *stable); (d) a coupling-adjusted* weakest-link *margin is sufficient (Cor. 2 and Prop. 4).*

This shows that the two aggregators play different roles in the same model. The additive form is the trace condition; the conjunctive form gives a certificate.

10. Lyapunov extension to time-varying systems

Consider $\dot{y} = [-D(t) + B(t)]y$ with $D(t) = \operatorname{diag}(k_1 a(t), k_2 b(t))$, $B_s = \frac{1}{2}(B + B^\top)$ and $m(t) = \min(k_1 a(t), k_2 b(t))$.

Theorem 11 (Uniform exponential stability). ESTABLISHED *If* $m(t) - \lambda_{\max}(B_s(t)) \geq \delta > 0$ *for all* $t \geq 0$, *then* $\|y(t)\| \leq e^{-\delta t}\|y(0)\|$.

Proof. Let $E = \frac{1}{2}y^\top y$. Since $y^\top B y = y^\top B_s y$, $\dot{E} = y^\top(-D + B_s)y \leq \big(-m(t) + \lambda_{\max}(B_s)\big)\|y\|^2 \leq -2\delta E$. Grönwall's inequality gives $E(t) \leq E(0)e^{-2\delta t}$. ∎

This motivates the model-specific coupling-adjusted margin

$$\boxed{M_{\text{dyn}}(t) = \min\big(k_1 a(t), k_2 b(t)\big) - \lambda_{\max}\big(B_s(t)\big)} \tag{18}$$

A uniformly positive M_{dyn} gives exponential contraction *in this model*. It is stronger than W and more restrictive in scope: k_1, k_2 and B must be estimated, and they are not S-I-C-T observables.

11. Counterexamples and fundamental limitations

11.1 Positive margin, unstable. $a = b = 1, k_1 = k_2 = 1, J = \left(\begin{smallmatrix} -1 & 2 \\ 2 & -1 \end{smallmatrix}\right)$: $W = 1$ but $\operatorname{spec}(J) = \{1, -3\}$.

11.2 Negative margin, stable. $a = -0.2, b = 1, J = \left(\begin{smallmatrix} 0.2 & -1 \\ 1 & -1 \end{smallmatrix}\right)$: $W = -0.2$, but $\operatorname{tr} J = -0.8$, $\det J = 0.8$, eigenvalues $-0.4 \pm 0.8i$.

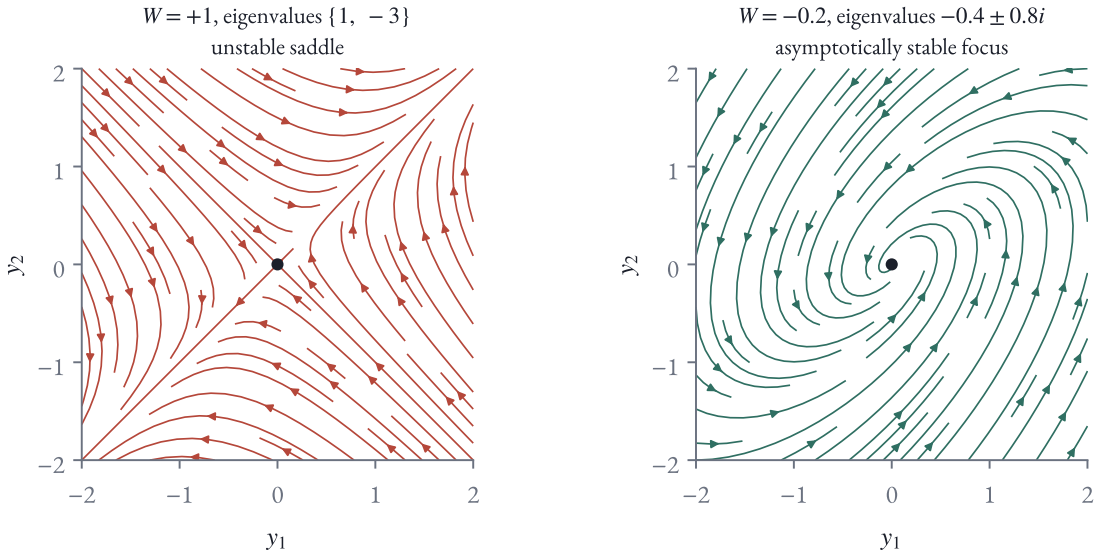

Figure 6. Computed phase portraits of the two counterexamples. Diagnostic margin and dynamical stability are logically independent.

11.3 Basin blindness. The draft argued this schematically. The construction below makes it exact.

Proposition 6 (Constructive basin blindness). CORRECTED ESTABLISHED *For* $s > 0$ *let* $\dot{z} = -U_s'(z)$ *with* $U_s(z) = \frac{1}{2}z^2 - z^3/(3s)$. *Every member has the attractor* $z^* = 0$ *with* $U_s''(0) = 1$: *identical linearizа-*

tion, recovery rate, stationary variance and lag-1 autocorrelation under small noise. Yet the escape barrier sits at $z = s$ with height $s^2/6$, which grows without bound in s.

Proof. $U_s'(z) = z(1 - z/s)$ vanishes at 0 (minimum) and s (maximum), and $U_s(s) = s^2/2 - s^2/3 = s^2/6$. ∎

So any indicator computed from local observables, including W_N and the classical critical-slowing-down statistics, cannot measure the perturbation size needed for a regime shift.

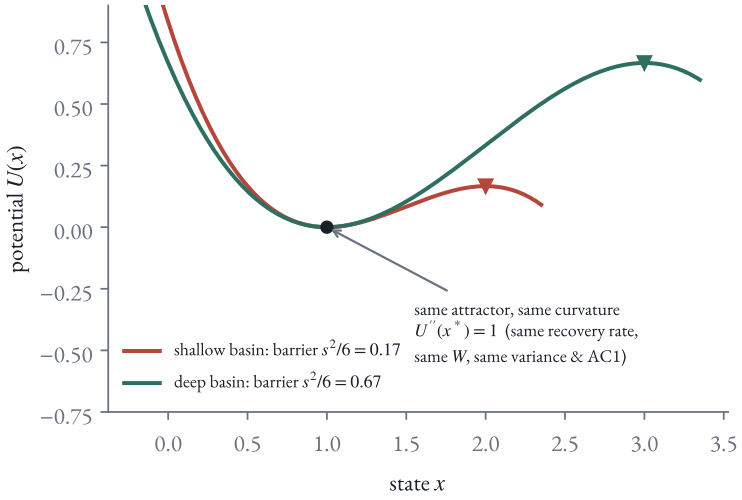

Figure 7. Two potentials with identical local behaviour at the attractor and a fourfold difference in barrier height.

11.4 Non-identifiability.

Proposition 7 (Observational non-identifiability). **ESTABLISHED** *Let (f_1, x_1) and (f_2, x_2) be two system–state pairs with $F(x_1) = F(x_2)$, where $F = (S, I, C, T)$, and with different stability classes. Then no deterministic function g of the four observations separates them, because $g(F(x_1)) = g(F(x_2))$.*

The statement is elementary, which is why it is a proposition rather than a theorem. Its consequence still matters: a four-number summary carries an information ceiling (see also Section 15.5).

12. Coupling without damping cannot stabilize

Let $\dot{z} = Az$ with $A = \left(\begin{smallmatrix} 0 & B \\ C & 0 \end{smallmatrix}\right)$ and $B, C \in \mathbb{R}^{n\times n}$. By the Schur complement,

$$\chi_A(q) = \det(q^2 I_n - BC). \tag{19}$$

Correction. Only for $n = 2$ does this expand to $q^4 - \operatorname{tr}(BC)\, q^2 + \det(BC)$; the draft omitted the block size. For every n the spectrum is symmetric under $q \mapsto -q$ (and $\operatorname{tr} A = 0$), so the origin is never asymptotically stable. Bounded neutral behaviour requires a purely imaginary, semisimple spectrum. With scalar damping, $\operatorname{spec}(A - \gamma I) = \operatorname{spec}(A) - \gamma$, and $A - \gamma I$ is Hurwitz if and only if $\gamma > \max_{\lambda \in \operatorname{spec} A} \operatorname{Re}\lambda$ (≥ 0).

Link to S-I-C-T. Setting $a = b = 0$ in (17) gives the case $n = 1$: eigenvalues $\pm\sqrt{uv}$, a saddle or a centre. Coupling alone never yields asymptotic stability. The diagonal damping supplied by positive capacity margins is *necessary*, which matches Prop. 5(a).

13. Stochastic dynamics and critical slowing down

For $dX_t = -\kappa X_t\,dt + \sigma\,dB_t$ with $\kappa > 0$, the stationary law has

$$\mathrm{Var}(X) = \frac{\sigma^2}{2\kappa}, \qquad \mathrm{Corr}(X_t, X_{t+\Delta}) = e^{-\kappa\Delta}. \tag{20}$$

As $\kappa \downarrow 0$, variance grows and lag-Δ autocorrelation tends to one. These are the textbook foundations of critical-slowing-down indicators, whose reach and limits are well documented [18, 9, 6, 7]. If a domain model independently establishes

$$\kappa = kW, \qquad k > 0, \tag{21}$$

then $\mathrm{Var}(X) = \sigma^2/(2kW)$ and $\rho_\Delta = e^{-kW\Delta}$. *Correction.* The bridge (21) is defined only for $W > 0$: for $W \le 0$ the process has no stationary distribution. It is an additional mechanistic assumption **HEURISTIC**, not a consequence of the S-I-C-T definitions. For $\kappa = 0.4$, $\sigma = 0.2$, $\Delta = 1$: recovery time 2.50, variance 0.050, lag-1 autocorrelation 0.670. A 4×10^5-step simulation reproduces 0.0500 and 0.6703.

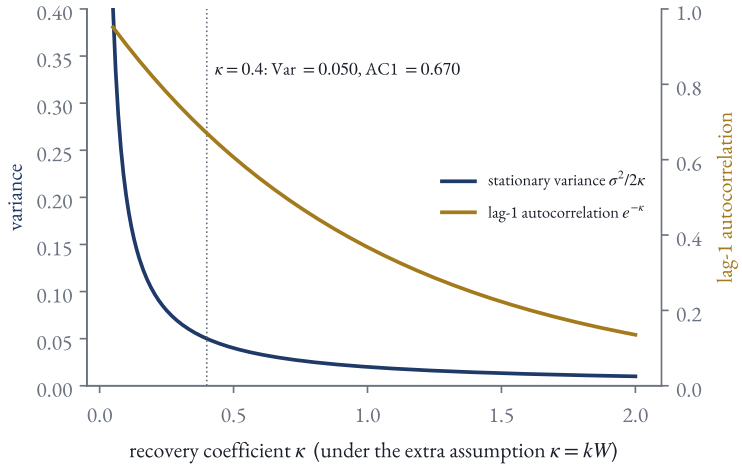

Figure 8. Variance and lag-1 autocorrelation of the Ornstein–Uhlenbeck model as the recovery coefficient falls. The S-I-C-T reading of the horizontal axis requires assumption (21).

14. An analytically grounded testbed: service networks

Take two measurable service channels: one processes incoming information requests, the other coordinates and executes the required state changes.

Coordinate	Measurement (common unit within each pair)
S	information-processing service rate (tasks/min)
I	information-request arrival rate (tasks/min)
C	coordination / execution service rate (operations/min)
T	required state-change arrival rate (operations/min)

For independent M/M/1 channels, $I < S$ and $T < C$ are exactly the conditions for each queue to have a stationary distribution, so $W_N > 0$ is exactly simultaneous stability. At equality the queue is null recurrent, not positive recurrent [13]. The result extends beyond independent channels:

Proposition 8 (Exactness for open Jackson networks). **ESTABLISHED** *In an open Jackson network with service rates μ_i and effective arrival rates λ_i solving the traffic equations, the network is positive recurrent if and only*

if $\lambda_i < \mu_i$ *for every node [11]. Hence stability* $\iff \min_i r_i > 0 \iff \max_i \rho_i < 1$, *with* $r_i = (\mu_i - \lambda_i)/(\mu_i + \lambda_i)$.

This covers the realistic tandem case, where coordination load is driven by information throughput. Here the conjunctive margin is the exact stability criterion for any number of channels, not a heuristic.

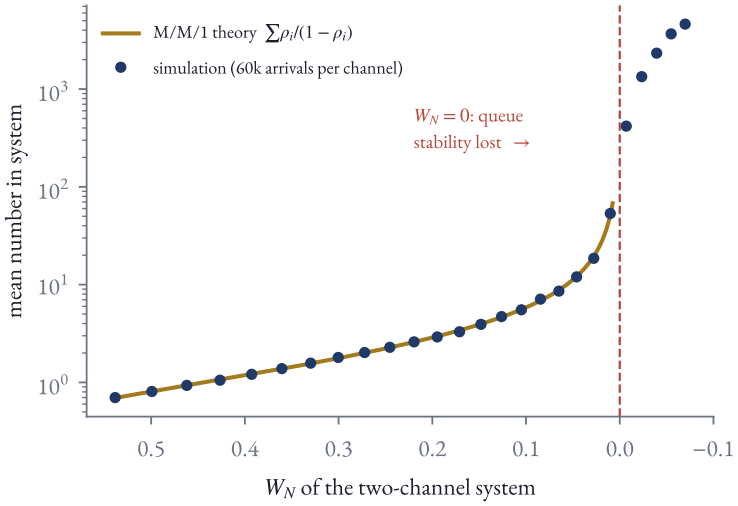

Figure 9. Two-channel M/M/1 testbed: mean number in system against W_N (simulation and theory). Stability is lost exactly at $W_N = 0$.

What the testbed can and cannot show. By Proposition 1, in a service network W_N is a monotone recoding of bottleneck utilization. The testbed is therefore a *positive control*. It validates instrumentation, error bounds (Props 2–3), M_{dyn} and transition timing, and it checks that S-I-C-T operationalizations recover a known answer. It cannot show predictive superiority over utilization metrics: as a single score, $\Delta\,\text{AUROC} = 0$ by construction. The scientifically open domains are those where capacity–load pairs are not already metered, such as organizational, socio-technical and multi-agent coordination failures.

15. Empirical Validation Protocol

The theorems establish internal consistency under explicit assumptions. They do not establish empirical value.

15.1 Preregistered hypotheses

Hypothesis	Testable claim
H1 — Diagnostic	Conjunctive margins identify independently defined capacity deficits. The outcome must not be defined as $W_N < 0$ (circularity).
H2 — Measurement design	S-I-C-T-guided choice of capacity–load pairs improves held-out prediction over models built from expert-chosen features of equal number, in domains without existing utilization metering. The baselines must include ρ^*.
H3 — Mechanistic	Independently measured margins predict perturbation recovery rates in specified models, i.e. test (21).
H4 — Transfer	A frozen operationalization keeps useful performance in held-out domains.
H5 — Robustness	Conclusions do not reverse under admissible pairwise rescaling or within the error bands of Props 2–3.

15.2 Benchmark systems

System	Validation target
Two-channel queue / Jackson network	Positive control (Section 14)
Noisy saddle-node bifurcation	Critical slowing down
Two-dimensional Ising lattice [17]	Continuous critical phenomena (not tipping)
Coupled nonlinear oscillators	Coordination and synchronization
Bistable potential family (Prop. 6)	Basin blindness
Rate-induced tipping [2]	Transformation load
Noise-induced switching; silent catastrophes [5]	Negative controls: failure without warning

Negative controls are essential. Not all critical transitions show critical slowing down, and an equilibrium phase transition is not a dynamical tipping event.

15.3 Baselines

Rolling variance and autocorrelation; recovery-rate estimates; **bottleneck utilization** ρ^*; the additive margin V; the individual margins r_S, r_C; a regularized model on all four raw coordinates; flexible nonlinear models on identical information; joint models with classical indicators and S-I-C-T features. An incremental-validity design must separate the value of *additional measurements* from the value of their *aggregation*.

15.4 Statistics

Let $Y_{t,h} = \mathbf{1}\{\text{transition in } (t, t+h]\}$, with h, the transition criterion, windows, preprocessing and model selection fixed in advance. With M_0 the baseline and M_1 the baseline plus S-I-C-T features, define $\Delta A = \mathrm{AUROC}(M_1) - \mathrm{AUROC}(M_0)$ and test $H_0\colon \Delta A \leq 0$ against $H_1\colon \Delta A > 0$. Report precision–recall under low prevalence, Brier score, calibration error, false-alarm rate, lead time and trajectory-level bootstrap intervals, with temporal splits to prevent leakage.

Pipeline canary. Compute $\mathrm{AUROC}(-W_N)$, $\mathrm{AUROC}(-L)$ and $\mathrm{AUROC}(\rho^*)$ on the same test set. By Proposition 1 they must agree to numerical precision. Any discrepancy means a bug or leakage. The suite reproduces the identity at 0.953831 for all three (test 15).

15.5 Information-theoretic limitation

Since $W_N = g(F)$ is deterministic, the data-processing inequality gives $\mathcal{I}(Y; W_N) \leq \mathcal{I}(Y; F)$ and $\mathcal{I}(Y; W_N \mid F) = 0$ [8]. Any advantage of W_N must come from inductive bias, interpretability, regularization or lower sample complexity, never from new information.

16. Falsification criteria

Test	Failure criterion
Proxy validity	Capacity and load cannot be measured commensurably in the target domain.
Incremental prediction	No meaningful held-out improvement over competitive baselines that include ρ^*.
Measurement design (H2)	S-I-C-T-chosen pairs do no better than expert-chosen features of equal number.
Mechanistic validity	Margin changes do not track measured recovery dynamics.
Transferability	Performance disappears under frozen out-of-domain mappings.
Robustness	Conclusions reverse under admissible scale changes or within error bands.
Causal validity	Controlled capacity interventions fail to produce preregistered effects.

Failing an empirical test does not touch the algebraic theorems. It limits the framework's relevance in the tested domain.

17. Open call for independent testing

Help test S-I-C-T — and try to break it

This framework stands or falls on evidence we cannot produce alone. Six kinds of help would change what we can claim:

1. **Proof reviewers** (mathematicians, control theorists, probabilists). Check Theorems 1–11 and Propositions 1–8, citing the number of any error. Prior-art pointers are especially welcome: if a result is already in the literature, we want to cite it.
2. **Replicators.** Run the suite in Appendix B (Python $\geq$ 3.10, NumPy, SciPy). The expected final line is `41 checks, 41 passed, 0 failed`. Send your platform and output. Ports to R or Julia are welcome.
3. **Counterexample hunters.** Find observation maps or dynamics where the stated hypotheses hold and a stated conclusion fails, or where the masking region of Equation (5) misleads in practice.
4. **Data partners.** Timestamped, paired capacity–load series with *independently* labelled transitions (incident logs, SRE and operations data, contact centres, hospital patient flow, supply chains, grid operations, agentic AI pipelines). As a guide, at least 50 labelled events, with measurement definitions fixed before analysis. Data can stay on your servers: we send code and preregistered analysis scripts.
5. **Testbed operators.** Run the two-channel service testbed (Section 14) as a positive control on a real multi-agent or service pipeline, with thresholds preregistered.
6. **Statisticians and methodologists.** Co-design the preregistration (power analysis, temporal validation, multiplicity control) for H1–H5.

Recognition. Every contribution is acknowledged in the next version. Substantive data or analysis contributions lead to co-authorship by mutual agreement. Null and negative results will be reported in full.
How to join. Email miklosroth@rothcomplexity.org with the subject line "S-I-C-T TEST — [role]". Include a short description of your expertise or data and, for replications, your output log. Project page: rothcomplexity.org/sict-framework-mathematical-validation.

18. Relationship to established theory, and novelty

Ashby's law of requisite variety [1] motivates matching regulatory capacity to demand; the inequalities here are not derived from it. Measurement theory restricts meaningful aggregation [14, 15]; the pairwise construction addresses one such restriction, not general cross-domain commensurability. Viability theory [3] and set invariance [16, 4] supply Theorem 9. Lyapunov and matrix theory [12, 10] supply Sections 9–10. Queueing theory [13, 11] supplies the testbed, and Proposition 1 shows that the normalized margin *is* the classical bottleneck ratio in disguise. Critical-transition research [18, 2, 6, 7, 9] supplies baselines and documented limits.

Novelty statement. We claim no new mathematics. The contribution is (a) a validated, explicitly conditional synthesis; (b) a clear separation of feasibility, invariance, stability and prediction; (c) the reduction of W_N to bottleneck utilization, which turns a vague superiority claim into a sharp, testable measurement-design hypothesis (H2); and (d) a preregistration-ready protocol with a built-in pipeline canary.

19. Conclusion

The reconstruction survives formal scrutiny with four repairs and no refutations. What survives: an exact representation of conjunctive constraints; algebraic identification of compensation masking; pairwise scale invariance; boundedness and monotonicity; robustness with observable and scale-free error bounds; a signed-distance geometry; a bounded smooth approximation; conditional positive invariance; conditional local and time-varying stability, in which additive margins are necessary and coupling-adjusted weakest-link margins are sufficient; and hard information-theoretic and identifiability limits. What does not survive: universality, unit-free raw aggregation, and any claim of predictive superiority that has not been tested against bottleneck utilization.

The most defensible description is a *conjunctive capacity–load diagnostic framework with conditional dynamical guarantees*. Whether its measurement design adds value in complex adaptive systems beyond service networks is now a sharp, falsifiable question. We invite you to help answer it (Section 17).

Data and code availability. No new empirical dataset is reported. The replication suite (`sict_validation.py`) is reproduced in full in Appendix B and is available from the author on request. **Research status.** Non-peer-reviewed theoretical working paper; independent mathematical review and preregistered empirical validation are required. **AI assistance disclosure.** Generative AI tools assisted mathematical reconstruction, validation code, counterexample construction, figure generation and manuscript preparation. The author is responsible for all claims, which remain subject to independent review. **Competing interests.** The author originated the S-I-C-T framework.

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A. Validation log

Output of `sict_validation.py` (seed 20261126). "NEW" marks checks of results introduced in this version.

#	Check	Detail	Result
1	T1 W≥0 ⇔ a≥0 and b≥0		PASS
2	Cor1 W≥0 ⇒ V≥0		PASS
3	Cor1 converse fails (masking region M non-empty)	share of $N(0,1)^2$ draws in M = 0.250 (theory 0.25)	PASS
4	Counterexample S=.9,I=.2,C=.3,T=.8 gives V=0.2, W=-0.5		PASS
5	Widget values a=.65,b=-.35 -> V=.30, W=-.35		PASS
6	T2 min(a,b) = (V-|a-b|)/2		PASS
7	T2' W≥0 ⇔ V≥|a-b|		PASS
8	T3 W_N in [-1,1]		PASS
9	T3 W_N≥0 ⇔ S≥I and C≥T		PASS
10	T4 pairwise scale invariance		PASS
11	T4 limit: NOT invariant under independent per-coordinate rescaling	sign flips in 19.7% of draws	PASS
12	Eq18 r = tanh(log(p/q)/2)		PASS
13	Eq18 W_N = tanh(L/2)		PASS
14	NEW W_N = (1-rho*)/(1+rho*), rho* = max(I/S, T/C)		PASS
15	NEW AUROC(-W_N) = AUROC(-L) = AUROC(rho*) exactly	AUROC = 0.953831 for all three	PASS
16	T5 dr/dp = 2q/(p+q)^2 ≥ 0		PASS
17	T5 dr/dq = -2p/(p+q)^2 ≤ 0		PASS
18	T6 |W_N^ - W_N| ≤ max(eS,eC)		PASS
19	NEW absolute raw error eta ⇒ |r^ - r| ≤ 2 eta/(p^+q^) (observable bound)		PASS
20	NEW relative raw error e ⇒ |r^ - r| ≤ -log(1-e) (scale-free bound)	bound = 0.1054 for e=10%	PASS
21	T7 W_N = signed l_inf distance to K=[0,inf)^2		PASS
22	T7 brute-force projection agrees		PASS
23	T8 WN - tau log2 ≤ W_tau < WN (tau=0.01)		PASS
24	T8 WN - tau log2 ≤ W_tau < WN (tau=0.15)		PASS
25	T8 WN - tau log2 ≤ W_tau < WN (tau=1.0)		PASS
26	Fig2 tau*log2 at tau=0.15 = 0.1040	0.10397	PASS
27	T10 Hurwitz ⇔ tr<0 and det>0		PASS
28	Cor2 W > rho/k_min ⇒ Hurwitz	1401 cases satisfied the condition	PASS
29	T11 (constant B) m - lambda_max(B_s) > 0 ⇒ Hurwitz		PASS
30	NEW Cor2 condition implies Lyapunov condition (Lyapunov is sharper)	3171 systems certified by Lyapunov but not by Gershgorin	PASS

#	Check	Detail	Result
31	T11 time-varying: ||y(t)|| ≤ e^{-delta t}||y(0)|| on a simulated system		PASS
32	11.1 W=+1 but spec = {-3, 1} (unstable)		PASS
33	11.2 W=-0.2 but tr=-0.8, det=0.8, Re(lambda)=-0.4 (stable)	eigenvalues -0.40+0.80j, -0.40-0.80j	PASS
34	Sec12 p(q)=q^4 - tr(BC) q^2 + det(BC) for 2x2 blocks		PASS
35	Sec12 undamped bipartite system never asymptotically stable		PASS
36	Sec12 general n: char. poly = det(q^2 I - BC) (n=3 test)		PASS
37	Fig5 OU: 1/kappa=2.50, Var=0.050, rho1=0.670	rho1 = 0.6703	PASS
38	Sec13 OU Monte Carlo matches Var and lag-1 autocorrelation	Var^=0.0500, rho1^=0.6703	PASS
39	Sec14.3 stable M/M/1 (I<S): mean number = rho/(1-rho) = 4	simulated 3.77	PASS
40	Sec14.3 overloaded M/M/1 (I>S): queue grows without bound	time-avg 295 after t=5000	PASS
41	Sec15.1 data-processing inequality I(Y;W) ≤ I(Y;F)	I(Y;W)=0.2034 nats, I(Y;F)=0.3555 nats	PASS

Summary: 41 checks, 41 passed, 0 failed.

B. Replication suite

Save as `sict_validation.py` and run `python3 sict_validation.py`. It runs in about two seconds on a laptop.

```
#!/usr/bin/env python3
"""
S-I-C-T reconstruction: replication & validation suite (v1.0, 2026-11-26)
Roth Complexity Lab — open for independent testing.

Every theorem, corollary, counterexample and numeric figure value in
"What Survives: A Validated Mathematical Reconstruction of the S-I-C-T
Framework" (Preprint v2.0, Nov 2026) is checked here,
either exactly (closed form) or by randomized falsification attempts.
A single FAIL line means a claim did not survive.

Run:  python3 sict_validation.py            (seed fixed for reproducibility)
Needs: Python >= 3.10, NumPy, SciPy.  Expected last line: "41 checks, 41 passed, 0 failed"
Report results, failures or counterexamples to miklosroth@rothcomplexity.org
(subject: "S-I-C-T TEST - replication").
"""
import numpy as np
from numpy.linalg import eigvals, eigvalsh

rng = np.random.default_rng(20261126)
N = 200_000
results = []

```

```
def check(name, ok, detail=""):
    results.append((name, bool(ok), detail))
    print(f"[{'PASS' if ok else 'FAIL'}] {name}" + (f"  — {detail}" if detail else ""))

def r(p, q):
    return (p - q) / (p + q)

# ---------------------------------------------------------------- T1 / Cor 1
a = rng.normal(size=N); b = rng.normal(size=N)
W = np.minimum(a, b); V = a + b
check("T1  W>=0  <=>  a>=0 and b>=0", np.all((W >= 0) == ((a >= 0) & (b >= 0))))
check("Cor1 W>=0 => V>=0", np.all(V[W >= 0] >= 0))
mask = (V >= 0) & (W < 0)
check("Cor1 converse fails (masking region M non-empty)", mask.mean() > 0,
      f"share of N(0,1)^2 draws in M = {mask.mean():.3f} (theory 0.25)")
S, I, C, T = 0.9, 0.2, 0.3, 0.8
check("Counterexample S=.9,I=.2,C=.3,T=.8 gives V=0.2, W=-0.5",
      np.isclose(S + C - I - T, 0.2) and np.isclose(min(S - I, C - T), -0.5))
check("Widget values a=.65,b=-.35 -> V=.30, W=-.35",
      np.isclose(0.65 - 0.35, 0.30) and np.isclose(min(0.65, -0.35), -0.35))

# ---------------------------------------------------------------- T2
check("T2  min(a,b) = (V-|a-b|)/2", np.allclose(W, (V - np.abs(a - b)) / 2))
check("T2' W>=0 <=> V>=|a-b|", np.all((W >= 0) == (V >= np.abs(a - b))))

# ---------------------------------------------------------------- T3 / T4
S, I, C, T = (rng.exponential(size=N) for _ in range(4))
rS, rC = r(S, I), r(C, T); WN = np.minimum(rS, rC)
check("T3  W_N in [-1,1]", np.all((WN >= -1) & (WN <= 1)))
check("T3  W_N>=0 <=> S>=I and C>=T", np.all((WN >= 0) == ((S >= I) & (C >= T))))
al, be = rng.lognormal(0, 2, N), rng.lognormal(0, 2, N)
check("T4  pairwise scale invariance", np.allclose(np.minimum(r(al*S, al*I), r(be*C, be*T)), WN))
g = [rng.lognormal(0, 1, N) for _ in range(4)]
WN_ind = np.minimum(r(g[0]*S, g[1]*I), r(g[2]*C, g[3]*T))
check("T4  limit: NOT invariant under independent per-coordinate rescaling",
      np.mean(np.sign(WN_ind) != np.sign(WN)) > 0.01,
      f"sign flips in {np.mean(np.sign(WN_ind) != np.sign(WN)):.1%} of draws")

# ---------------------------------------------------------------- 4.1 tanh identity / rank equivalence
L = np.minimum(np.log(S / I), np.log(C / T))
check("Eq18 r = tanh(log(p/q)/2)", np.allclose(rS, np.tanh(np.log(S / I) / 2)))
check("Eq18 W_N = tanh(L/2)", np.allclose(WN, np.tanh(L / 2)))
rho_star = np.maximum(I / S, T / C)
check("NEW W_N = (1-rho*)/(1+rho*), rho* = max(I/S, T/C)",
      np.allclose(WN, (1 - rho_star) / (1 + rho_star)))

def auroc(score, y):
    """Mann-Whitney AUROC (higher score = more likely y=1); ties averaged."""
    from scipy.stats import rankdata
    rk = rankdata(score); n1 = y.sum(); n0 = len(y) - n1
    return (rk[y == 1].sum() - n1 * (n1 + 1) / 2) / (n1 * n0)

y = (rng.random(N) < 1 / (1 + np.exp(-3 * (rho_star - 1)))).astype(int)
A_WN, A_L, A_rho = auroc(-WN, y), auroc(-L, y), auroc(rho_star, y)
check("NEW AUROC(-W_N) = AUROC(-L) = AUROC(rho*) exactly",
      np.isclose(A_WN, A_L) and np.isclose(A_WN, A_rho), f"AUROC = {A_WN:.6f} for all three")

# ---------------------------------------------------------------- T5
p, q = rng.exponential(size=N), rng.exponential(size=N); h = 1e-6
```

```
dp = (r(p + h, q) - r(p - h, q)) / (2 * h); dq = (r(p, q + h) - r(p, q - h)) / (2 * h)
check("T5  dr/dp = 2q/(p+q)^2 >= 0", np.allclose(dp, 2 * q / (p + q) ** 2, rtol=1e-4) and np.all(dp >= 0))
check("T5  dr/dq = -2p/(p+q)^2 <= 0", np.allclose(dq, -2 * p / (p + q) ** 2, rtol=1e-4) and np.all(dq <= 0))

# ---------------------------------------------------------------- T6 + new error propagation
eS, eC = rng.uniform(0, .3, N), rng.uniform(0, .3, N)
rSh = rS + rng.uniform(-1, 1, N) * eS; rCh = rC + rng.uniform(-1, 1, N) * eC
check("T6  |W_N^ - W_N| <= max(eS,eC)",
      np.all(np.abs(np.minimum(rSh, rCh) - WN) <= np.maximum(eS, eC) + 1e-12))
eta = 0.05
ph = p + rng.uniform(-eta, eta, N); qh = q + rng.uniform(-eta, eta, N)
ok = (ph + qh) > 0
bound = 2 * eta / (ph + qh)
check("NEW absolute raw error eta  =>  |r^ - r| <= 2 eta/(p^+q^)  (observable bound)",
      np.all(np.abs(r(ph, qh) - r(p, q))[ok] <= bound[ok] + 1e-12))
er = 0.10
ph = p * (1 + rng.uniform(-er, er, N)); qh = q * (1 + rng.uniform(-er, er, N))
check("NEW relative raw error e  =>  |r^ - r| <= -log(1-e)  (scale-free bound)",
      np.all(np.abs(r(ph, qh) - r(p, q)) <= -np.log(1 - er) + 1e-12), f"bound = {-np.log(1-er):.4f} for e=10%")

# ---------------------------------------------------------------- T7 signed distance (l_inf)
z = rng.uniform(-1, 1, (N, 2)); inK = np.all(z >= 0, axis=1)
d_in = np.min(z, axis=1)                                  # dist to complement
d_out = np.max(np.maximum(-z, 0), axis=1)                 # dist to K
signed = np.where(inK, d_in, -d_out)
check("T7  W_N = signed l_inf distance to K=[0,inf)^2", np.allclose(signed, np.min(z, axis=1)))
# brute-force check on a subsample: projection onto K in l_inf and distance to K^c
zs = z[:2000]; best = []
for v in zs:
    if np.all(v >= 0):
        best.append(min(v))      # move the smaller coordinate just below 0
    else:
        proj = np.maximum(v, 0); best.append(-np.max(np.abs(v - proj)))
check("T7  brute-force projection agrees", np.allclose(best, np.min(zs, axis=1)))

# ---------------------------------------------------------------- T8 soft-min
for tau in [0.01, 0.15, 1.0]:
    Wt = -tau * np.log(np.exp(-rS / tau) + np.exp(-rC / tau))
    check(f"T8  WN - tau log2 <= W_tau < WN  (tau={tau})",
          np.all(Wt <= WN + 1e-12) and np.all(Wt >= WN - tau * np.log(2) - 1e-12))
check("Fig2 tau*log2 at tau=0.15 = 0.1040", np.isclose(0.15 * np.log(2), 0.1040, atol=5e-5),
      f"{0.15*np.log(2):.5f}")

# ---------------------------------------------------------------- T10 Routh-Hurwitz (2x2)
M = 50_000
k1, k2 = rng.uniform(.1, 3, M), rng.uniform(.1, 3, M)
aa, bb = rng.normal(0, 1, M), rng.normal(0, 1, M)
u, v = rng.normal(0, 1.5, M), rng.normal(0, 1.5, M)
J = np.stack([np.stack([-k1 * aa, u], -1), np.stack([v, -k2 * bb], -1)], -2)
maxre = np.max(eigvals(J).real, axis=1)
hur = maxre < 0
rh = (k1 * aa + k2 * bb > 0) & (k1 * k2 * aa * bb - u * v > 0)
check("T10 Hurwitz <=> tr<0 and det>0", np.all(hur == rh))

# ---------------------------------------------------------------- Cor 2 (Gershgorin) & T11 (Lyapunov)
Wraw = np.minimum(aa, bb); rho = np.maximum(np.abs(u), np.abs(v)); kmin = np.minimum(k1, k2)
g_cond = Wraw > rho / kmin
```

```
check("Cor2 W > rho/k_min  =>  Hurwitz", np.all(hur[g_cond]), f"{g_cond.sum()} cases satisfied the condition")
m = np.minimum(k1 * aa, k2 * bb); lam = np.abs(u + v) / 2      # lambda_max of sym. part of [[0,u],[v,0]]
l_cond = m - lam > 0
check("T11 (constant B) m - lambda_max(B_s) > 0  =>  Hurwitz", np.all(hur[l_cond]))
check("NEW Cor2 condition implies Lyapunov condition (Lyapunov is sharper)",
      np.all(l_cond[g_cond]) and (l_cond & ~g_cond).sum() > 0,
      f"{(l_cond & ~g_cond).sum()} systems certified by Lyapunov but not by Gershgorin")

# T11 time-varying: simulate and check the exponential envelope
def simulate(delta_target=0.2, Tend=20.0, dt=1e-3):
    t = 0.0; yv = np.array([1.0, -0.7]); ok = True; y0 = np.linalg.norm(yv)
    while t < Tend:
        a_t, b_t = 1.0 + 0.5 * np.sin(t), 1.2 + 0.4 * np.cos(1.3 * t)
        Bt = np.array([[0.0, 0.6 * np.sin(2 * t)], [-0.3 + 0.2 * np.cos(t), 0.0]])
        Bs = (Bt + Bt.T) / 2
        mt = min(a_t, b_t); margin = mt - eigvalsh(Bs).max()
        ok &= margin >= delta_target
        A = -np.diag([a_t, b_t]) + Bt
        k1_ = A @ yv; k2_ = A @ (yv + dt / 2 * k1_); k3_ = A @ (yv + dt / 2 * k2_); k4_ = A @ (yv + dt * k3_)
        yv = yv + dt / 6 * (k1_ + 2 * k2_ + 2 * k3_ + k4_); t += dt
        if np.linalg.norm(yv) > np.exp(-delta_target * t) * y0 * (1 + 1e-6):
            return False, ok
    return True, ok
env_ok, cond_ok = simulate()
check("T11 time-varying: ||y(t)|| <= e^{-delta t}||y(0)|| on a simulated system", env_ok and cond_ok)

# ------------------------------------------------------------------ 11.1 / 11.2 counterexamples
ev1 = np.sort(eigvals(np.array([[-1, 2], [2, -1]])).real)
check("11.1 W=+1 but spec = {-3, 1} (unstable)", np.allclose(ev1, [-3, 1]))
J2 = np.array([[0.2, -1], [1, -1]]); ev2 = eigvals(J2)
check("11.2 W=-0.2 but tr=-0.8, det=0.8, Re(lambda)=-0.4 (stable)",
      np.isclose(np.trace(J2), -0.8) and np.isclose(np.linalg.det(J2), 0.8) and np.allclose(ev2.real, -0.4),
      f"eigenvalues {ev2[0]:.2f}, {ev2[1]:.2f}")

# ------------------------------------------------------------------ Sec. 12 bipartite
ok_poly, ok_sym = True, True
for _ in range(2000):
    B, Cm = rng.normal(size=(2, 2)), rng.normal(size=(2, 2))
    A = np.block([[np.zeros((2, 2)), B], [Cm, np.zeros((2, 2))]])
    P, Q = np.trace(B @ Cm), np.linalg.det(B @ Cm)
    lam_ = eigvals(A)
    rts = np.roots([1, 0, -P, 0, Q])
    ok_poly &= (np.abs(lam_**4 - P * lam_**2 + Q).max() < 1e-8) and \
               (max(min(abs(rts - l)) for l in lam_) < 1e-5)   # match roots by distance, not by sort order
    ok_sym &= np.max(lam_.real) >= -1e-9
check("Sec12 p(q)=q^4 - tr(BC) q^2 + det(BC) for 2x2 blocks", ok_poly)
check("Sec12 undamped bipartite system never asymptotically stable", ok_sym)
B3, C3 = rng.normal(size=(3, 3)), rng.normal(size=(3, 3))
A3 = np.block([[np.zeros((3, 3)), B3], [C3, np.zeros((3, 3))]])
check("Sec12 general n: char. poly = det(q^2 I - BC) (n=3 test)",
      np.allclose(np.sort_complex(eigvals(A3) ** 2), np.sort_complex(np.repeat(eigvals(B3 @ C3), 2)), atol=1e-6))

# ------------------------------------------------------------------ Sec. 13 OU
kap, sig, Dl = 0.40, 0.2, 1.0
```

```
check("Fig5 OU: 1/kappa=2.50, Var=0.050, rho1=0.670",
      np.isclose(1 / kap, 2.5) and np.isclose(sig**2 / (2 * kap), 0.05) and np.isclose(np.exp(-kap * Dl), 0.670, atol=5e-4),
      f"rho1 = {np.exp(-kap*Dl):.4f}")
# Monte Carlo check of stationary variance and lag-1 autocorrelation (exact discretisation)
n = 400_000; x = np.empty(n); x[0] = 0.0
phi = np.exp(-kap * Dl); s = sig * np.sqrt((1 - phi**2) / (2 * kap))
eps = rng.normal(size=n)
for i in range(1, n):
    x[i] = phi * x[i - 1] + s * eps[i]
x = x[1000:]
check("Sec13 OU Monte Carlo matches Var and lag-1 autocorrelation",
      abs(x.var() - 0.05) < 0.002 and abs(np.corrcoef(x[:-1], x[1:])[0, 1] - phi) < 0.01,
      f"Var^={x.var():.4f}, rho1^={np.corrcoef(x[:-1], x[1:])[0,1]:.4f}")

# ------------------------------------------------------------------ Sec. 14.3 queueing
def mm1_mean_queue(lam, mu, horizon=20_000.0):
    """Gillespie simulation of an M/M/1 queue; returns time-average number in system."""
    t, nq, area = 0.0, 0, 0.0
    while t < horizon:
        rate = lam + (mu if nq > 0 else 0.0)
        dt = rng.exponential(1 / rate); area += nq * dt; t += dt
        nq += 1 if rng.random() < lam / rate else -1
    return area / t
lam, mu = 0.8, 1.0
Lq = mm1_mean_queue(lam, mu)
check("Sec14.3 stable M/M/1 (I<S): mean number = rho/(1-rho) = 4", abs(Lq - 4.0) < 0.6, f"simulated {Lq:.2f}")
Lq_over = mm1_mean_queue(1.1, 1.0, horizon=5000.0)
check("Sec14.3 overloaded M/M/1 (I>S): queue grows without bound", Lq_over > 100, f"time-avg {Lq_over:.0f} after t=5000")

# ------------------------------------------------------------------ Sec. 15.1 information
# Discrete example: I(Y;g(F)) <= I(Y;F)
def mi(xd, yd):
    xs, xi = np.unique(xd, return_inverse=True); ys, yi = np.unique(yd, return_inverse=True)
    joint = np.zeros((len(xs), len(ys))); np.add.at(joint, (xi, yi), 1); joint /= joint.sum()
    px, py = joint.sum(1, keepdims=True), joint.sum(0, keepdims=True)
    nz = joint > 0
    return float((joint[nz] * np.log(joint[nz] / (px @ py)[nz])).sum())
Fd = rng.integers(0, 4, (N, 4)); Wd = np.minimum(Fd[:, 0] - Fd[:, 1], Fd[:, 2] - Fd[:, 3])
Yd = ((Fd[:, 0] - Fd[:, 1] + 0.5 * (Fd[:, 2] - Fd[:, 3]) + rng.normal(0, 1, N)) < 0).astype(int)
Fkey = Fd @ np.array([64, 16, 4, 1])
check("Sec15.1 data-processing inequality I(Y;W) <= I(Y;F)", mi(Wd, Yd) <= mi(Fkey, Yd) + 1e-12,
      f"I(Y;W)={mi(Wd,Yd):.4f} nats, I(Y;F)={mi(Fkey,Yd):.4f} nats")

# ------------------------------------------------------------------ summary
n_fail = sum(not ok for _, ok, _ in results)
print(f"\n{len(results)} checks, {len(results)-n_fail} passed, {n_fail} failed")
```